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The geometry of the entropic principle and the shape of the universe

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Abstract: Ooguri, Vafa, and Verlinde have outlined an approach to two-dimensional accelerating string cosmology which is based on topological string theory, the ultimate objective being to develop a string-theoretic understanding of "creating the Universe from nothing". The key technical idea here is to assign *two different* Lorentzian spacetimes to a certain Euclidean space. Here we give a simple framework which allows this to be done in a systematic way. This framework can be extended to higher dimensions. We find then that the general shape of the spatial sections of the newly created Universe is constrained by the OVV formalism: the sections have to be flat and compact.

Keywords: Topological Strings, dS vacua in string theory, Differential and Algebraic Geometry.

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Contents

1. The cosmology of topological strings

Ooguri, Vafa, and Verlinde have put forward [1] [see also [2]] a novel approach to quantum cosmology, one in which the topological string partition function is related to the wave function of a two-dimensional Universe [3] in a mini-superspace description. One thus obtains a "Hartle-Hawking wave function for flux compactifications". *This opens up the prospect of a string-theoretic account of the creation of the Universe from "nothing"* [4][5]. The hope is that, if these ideas can be made to work in four dimensions, it will be possible to construct a much-needed string vacuum selection principle[1]. In particular, Ooguri et al are interested in using their wave function to constrain the initial *geometry* of the newly created Universe[2].

Even at the classical or semi-classical levels, the techniques of Ooguri et al have some very unusual features. In this note we explain the geometric meaning of one of the crucial innovations in [1], and use this explanation as a guide to what might be involved in extending these methods to Universes with dimensions higher than two. We also point out that, even without a detailed understanding of the OVV wave function in the higher-dimensional case, we can use self-consistency conditions to draw conclusions about the nature of the initial geometry predicted by that wave function.

[1]See [6][7][8][9].

[2]Another attempt to achieve this goal, based instead on the ideas of [6], is described in [10].

JHEP10(2006)029

Ooguri et al [1] work with a compactification of IIB string theory to Euclidean two-dimensional anti-de Sitter space, the hyperbolic space H^2 of curvature $-1/L^2$; they use the foliation of H^n, familiar from studies of the AdS/CFT correspondence, by *flat* slices [11]. However — and this is crucial for the cosmological application — they then perform a further compactification to a space with geometry $H^2/\mathbb{Z}$ and topology $\mathbb{R} \times S^1$, where S^1 is a circle. The metric is

$$g(H^2/\mathbb{Z})_{++} \;=\; K^2\, e^{(2\,\rho/L)}\, d\tau^2 \;+\; d\rho^2; \tag{1.1}$$

here, τ is an angular coordinate on a circle with radius K at $\rho = 0$. Notice that this can easily be generalized to a simple "partial compactification" of H^{n+1} for any n, of the form $H^{n+1}/\mathbb{Z}^n$, with topology $\mathbb{R} \times T^n$, where T^n is the n-torus.

The crux of the Ooguri et al construction is the remarkable claim that this Euclidean space can be interpreted in *two different ways*, as follows. From the point of view of the topological string it corresponds to a local anti-de Sitter geometry; τ is "Euclidean time", leading to a Witten index which counts the degeneracy of ground states of a certain black hole configuration. But Ooguri et al *also* interpret this geometry as a sort of Euclidean version of de Sitter spacetime. The idea here is that ρ is *equally entitled* to be regarded as some kind of "Euclidean time" — *there is no way to distinguish one coordinate as "time"* on a Euclidean manifold. The manifold with metric given in equation (1.1), with its circular [toral] sections which "expand" exponentially as ρ increases, then somehow corresponds to a Lorentzian accelerating universe, and this is how the topological string partition function makes contact with cosmology.

Of course, as Ooguri et al themselves point out, this second interpretation cannot be taken literally within the usual Hartle-Hawking formalism, since complexifying ρ in the usual manner of Euclidean quantum gravity would *complexify the metric itself*; so the "correspondence" of equation (1.1) with de Sitter spacetime is obscure.

Nevertheless the idea that $H^2/\mathbb{Z}$ could have a *double interpretation* is the key device which, in [1], is supposed to implement the connection between the topological string and the wave function of the Universe. Furthermore, the basic proposal that both ρ and τ are equally entitled to be regarded as "Euclidean time" is clearly reasonable. We see that, in order for the Ooguri et al programme to proceed, we have to answer the first of two basic questions: *how can we make sense of the intuition that the metric in equation (1.1) somehow corresponds to an accelerating cosmology?* We can formulate this question more concretely as: how is it possible for there to be two distinct *complexifications* [one AdS-like, the other dS-like] of $H^2/\mathbb{Z}$?

In the course of investigating this question, we soon realize that the two-dimensional situation considered by Ooguri et al is very special, because in that dimension the symmetry group of (n+1)-dimensional anti-de Sitter spacetime, O(2,n), is isomorphic to the symmetry group of (n+1)-dimensional de Sitter spacetime, O(n+1,1). From this point of view, the double interpretation of $H^2/\mathbb{Z}$ *must* be valid in some sense in the two-dimensional case. For if the de Sitter group is the same as the anti-de Sitter group, and granted that H^2 can be analytically continued to AdS_2, then there *must* be some natural way of associating [some version of] H^2 with dS_2. But O(2,n) is certainly not isomorphic to O(n+1,1) for higher n,

JHEP10(2006)029

and this immediately raises our second question: does the double interpretation of $H^2/\mathbb{Z}$ *only* work in two dimensions?

Our tactic for dealing with these questions focuses on a simple ambiguity in the procedure of *complexification*. The ambiguity arises partly from the apparently trivial observation that, at least for orientable spacetimes, the $(- \ + \ + +)$ signature is in no way preferable to $(+ \ - \ - -)$ signature, and partly from the observation of Ooguri et al that there is no unique way of deciding how to define Euclidean "time".

Because of its traditional association with the "no-boundary" proposal in Euclidean quantum gravity, the process of complexification is usually held to lead *uniquely* from the four-sphere to [global] de Sitter spacetime. We begin with a critical review of this idea. We show that there is a sense in which a *local* anti-de Sitter metric can also be obtained by complexifying the sphere. However, this construction leads to a spacetime in which the spatial sections immediately contract after the universe is created, and so the claim that complexification leads uniquely from the sphere to a spacetime *in which the spatial sections eventually reach macroscopic size* can be justified. The ideas of Ooguri et al prompt us to investigate whether an analogous claim is justified in the case of hyperbolic space: does complexification lead uniquely to AdS? The answer is no: one can obtain [several physically acceptable versions of] de Sitter spacetime in this way. Using these ideas, we give a concrete interpretation of the double interpretation of $H^2/\mathbb{Z}$ needed for the arguments of Ooguri et al.

When we attempt to extend the construction to higher dimensions, we find that it only makes sense for cosmological models with flat [toral] spatial sections. Thus the *shape* of the Universe is "emergent" in this formalism, in precisely the same way [as has recently been argued by Hartle [12]] that Lorentzian signature emerges from the Hartle-Hawking wave function. It remains to be seen whether the initial *size* likewise "emerges" from the OVV wave function.

2. Complexification ambiguity: the case of the sphere

Complexification is a way of assigning a Lorentzian manifold to a Euclidean one. It has been applied to curved spacetimes in at least *two distinct ways*.

First, it appeared as a way of studying the remote past of de Sitter spacetime: in the Hartle-Hawking approach, one uses complexification to convert the contracting half of de Sitter spacetime to half of a four-sphere S^4, which remains attached to the [still Lorentzian] upper half of dS_4. (Here we refer to "SSdS", the Spatially Spherical version of de Sitter spacetime; see below.)

The second major application of complexification was to string theory, in the form of the AdS/CFT correspondence [11][13]. Usually "AdS" here means H^4, the hyperbolic space, from which AdS_4 can be obtained in this way. The work of Ooguri et al essentially constructs the analogue of the Hartle-Hawking wave function, defined on a hyperbolic space instead of a sphere. We therefore need to ask precisely how complexification works for hyperbolic space. As a preparation for that, let us review the spherical case in the light of the observations made in [1].

JHEP10(2006)029

It is a basic fact that if one takes the standard metric on the four-sphere of radius L,

$$g(\mathrm{S}^4)_{++++} \;=\; \mathrm{L}^2 \left\{ \mathrm{d}\xi^2 \;+\; \cos^2(\xi)\,[\mathrm{d}\chi^2 \;+\; \sin^2(\chi)\{\mathrm{d}\theta^2 \;+\; \sin^2(\theta)\,\mathrm{d}\phi^2\}] \right\}, \qquad (2.1)$$

where all of the coordinates are angular, and continues $\xi \rightarrow \mathrm{iT/L}$, then the result is the *global* de Sitter metric, in its Spatially Spherical form,

$$g(\mathrm{SSdS}_4)_{-+++} \;=\; -\,\mathrm{dT}^2 \;+\; \mathrm{L}^2\cosh^2(\mathrm{T/L})\,[\mathrm{d}\chi^2 \;+\; \sin^2(\chi)\{\mathrm{d}\theta^2 \;+\; \sin^2(\theta)\,\mathrm{d}\phi^2\}], \qquad (2.2)$$

with the indicated signature.

Motivated by the arguments of Ooguri et al [1], we now observe that ξ and χ have the same status as angles, and the same range; *there is no justification for preferring one to the other.* Let us complexify χ in equation (2.1) instead of ξ, replacing $\chi \rightarrow \pm\mathrm{is/L}$, and for convenience relabelling ξ as u/L [without complexifying it]. We obtain, since $\sin^2(\chi)$ reverses sign under complexification of χ,

$$g(\mathrm{DAdS}_4)_{+---} \;=\; \mathrm{du}^2 \;-\; \cos^2(\mathrm{u/L})\,[\mathrm{ds}^2 \;+\; \mathrm{L}^2\sinh^2(\mathrm{s/L})\{\mathrm{d}\theta^2 \;+\; \sin^2(\theta)\,\mathrm{d}\phi^2\}]. \qquad (2.3)$$

But, *purely locally*, this is the *anti*-de Sitter metric, in $(+\,-\,-\,-)$ signature, and expressed in terms of coordinates [14][15][16] based on the timelike geodesics which are perpendicular to the spatial sections; the coordinate u is proper time along these geodesics. These coordinates do not cover the entire spacetime, of course, because these timelike geodesics intersect, being drawn together by the attractive nature of gravity in anti-de Sitter spacetime [which satisfies the Strong Energy Condition]. This is why these coordinates give the false impression that there is no timelike Killing vector in AdS — there is one, but it does not correspond to the time coordinate u. On the other hand, these coordinates do have the virtue of reflecting the behaviour of inertial observers in AdS_4. In fact, these coordinates cover the Cauchy development of a single spacelike slice: in this sense they are the precise analogues of the standard global coordinates in dS_4, which happen to cover the entire spacetime simply because gravity is repulsive in that case, ensuring that the worldlines of inertial observers do not intersect.

The fact nevertheless remains that in continuing (2.1) to (2.3) we have only continued S^4 to a small *part* of AdS_4; see [17] for a detailed discussion of related issues. This part is of course extensible; objects can leave or enter the spacetime without encountering any singularity. This does not make sense physically, *particularly in the context of "creation from nothing"*; for if objects or signals can enter the spacetime along a null surface, it is doubtful that one can claim that the Universe was "created" on a specific spacelike surface.

Fortunately there is an extremely natural way to solve this problem, as follows. The Lorentzian metric (2.3) is of course a purely local structure. We are not told how to select the global structure from the large range of possibilities compatible with this local metric. In particular, the spatial sections here have the geometry of three-dimensional hyperbolic space. These can be compactified: that is, we interpret the spatial part of the metric as a metric on a space of the form H^3/Γ, where Γ is some discrete freely acting infinite group of H^3 isometries such that the quotient is compact. The compactified spacetime is incomplete only in the past, not along null surfaces, and so it makes sense to speak of it being created

JHEP10(2006)029

along a spacelike surface. We shall discuss this in more detail below. For the present we merely note that, with this interpretation, *one loses the global timelike Killing vector* defined on full AdS_4: it does not project to a Killing vector on the quotient. The metric in (2.3) is a genuinely *dynamic* metric on a spacetime, with topology $\mathbb{R} \times (\mathbb{R}^3/\Gamma)$, with spatial sections which expand from zero size and then contract back to zero size[3]. We can call this "Dynamic AdS_4" (hence the notation in (2.3)); *the timelike symmetry has been broken topologically.* It follows that this spacetime is physically distinct from true AdS_4.

Thus, the alternative complexification of S^4 does not lead to AdS_4, but rather to a spacetime with compact spatial sections with a non-trivial evolution controlled by the metric (2.3). If this spacetime is "created from nothing" along its spacelike surface of zero extrinsic curvature at $u = 0$, then it will immediately begin to contract; thus it will never reach macroscopic size.

We conclude that there is a "complexification ambiguity" for the sphere, in the sense that the sphere can indeed be continued to two distinct Lorentzian spacetimes. But one of these continuations fails to attain macroscopic size, so we have a concrete justification for discarding it. [This argument is modelled on Hartle's [12] discussion of the "emergence" of Lorentzian signature; see below.] In this sense, we can continue to claim that there is only one physically significant continuation of the sphere.

Nevertheless, there are some interesting lessons here. First, it is clear more generally that Euclidean spaces will sometimes have more than one Lorentzian version if we accept both $(+ - - -)$ and $(- + + +)$ signatures. Actually, for the most general topologically non-trivial spacetimes, the two possible choices of signature *are not fully physically equivalent*, a surprising fact first pointed out by Carlip and DeWitt-Morette [18]. This shows that the distinction we are discussing here is by no means trivial. Nevertheless, none of the issues raised in [18] actually arise here — all of our spacetimes, including the topologically non-trivial ones like Dynamical AdS, are orientable in all of the possible senses — so, in our case, there can be no initial justification for preferring one signature to the other. We merely propose to take this observation seriously when applying complexification.

A second lesson to be drawn from the discussion above is that one should bear in mind that *the sign of the curvature of a Lorentzian manifold depends on the convention for signature.* Thus anti-de Sitter spacetime is a spacetime of *positive* curvature in $(+ - - -)$ signature, while de Sitter spacetime has *negative* curvature in that convention. Hence the association of a *positively* curved version of AdS_4 with the sphere is perhaps not so surprising. This of course opens the way to justifying the hope of Ooguri et al, that some version of de Sitter spacetime can emerge from their basic negatively curved Euclidean space.

In this section we have seen that one has to qualify the claim that de Sitter spacetime is *the only* Lorentzian continuation of the four-sphere: one can also obtain a *local* version of anti-de Sitter spacetime in this way. Thus complexification is "locally ambiguous" in this sense. However, *in the case of the sphere,* global considerations effectively remove the

[3]The initial and final points do not correspond to curvature singularities, but generically they would: this geometry fails to satisfy the *generic condition* given on page 266 of [14]. The singularity theorems imply that the slightest generic perturbation causes these points to become genuinely singular.

JHEP10(2006)029

ambiguity, since we saw that one certainly cannot obtain *global* anti-de Sitter spacetime by complexifying the sphere: instead one obtains a spacetime which is no sooner created than it shrinks to non-existence. In this sense, the usual understanding is correct: de Sitter spacetime is indeed the unique *macroscopic* complexification of the sphere.

The situation becomes more interesting in the case of hyperbolic geometry, however, as we shall now see.

3. Complexification ambiguity: the case of hyperbolic space

The hyperbolic Euclidean space H^4, with its metric of constant curvature equal to $-1/\mathrm{L}^2$, can be defined as a connected component of the locus

$$-\,\mathrm{A}^2 \;+\; \mathrm{B}^2 \;+\; \mathrm{X}^2 \;+\; \mathrm{Y}^2 \;+\; \mathrm{Z}^2 \;=\; -\,\mathrm{L}^2, \tag{3.1}$$

defined in a five-dimensional Minkowski space. It is clear that all of the coordinates except A can range in $(-\infty,\ +\infty)$, while A has to satisfy $\mathrm{A}^2 \;\geq\; \mathrm{L}^2$. We always pick the connected component on which A is positive.

Hyperbolic space can be globally foliated in a variety of interesting ways. The leaves of the foliation are distinguished by a parameter; *this parameter plays the role of Euclidean time.* Clearly there is no "preferred" foliation. This statement is the generalized version of the observation, made by Ooguri et al, that both ρ and τ are equally entitled to be interpreted as "time" in equation (1.1).

We shall now discuss four physically interesting ways of foliating H^4, and their various complexifications.

3.1 Foliation corresponding to anti-de Sitter

The underlying manifold of H^4 can be regarded as the interior of a ball, as shown in Figure 1; this is clear if we write equation (3.1) as

$$\mathrm{B}^2 \;+\; \mathrm{X}^2 \;+\; \mathrm{Y}^2 \;+\; \mathrm{Z}^2 \;=\; \mathrm{A}^2 \;-\; \mathrm{L}^2, \tag{3.2}$$

since we clearly have a three-sphere at infinity. The conformal boundary is also shown, but points on the boundary are of course not points of H^4.

Choosing the connected component on which [in equation (3.1)] A is positive, we can pick coordinates Ψ,Σ,θ,ϕ such that

$$\begin{aligned}
\mathrm{A} &= \mathrm{L}\,\cosh(\Psi)\,\cosh(\Sigma)\\
\mathrm{B} &= \mathrm{L}\,\sinh(\Psi)\,\cosh(\Sigma)\\
\mathrm{Z} &= \mathrm{L}\,\sinh(\Sigma)\,\cos(\theta)\\
\mathrm{Y} &= \mathrm{L}\,\sinh(\Sigma)\,\sin(\theta)\,\cos(\phi)\\
\mathrm{X} &= \mathrm{L}\,\sinh(\Sigma)\,\sin(\theta)\,\sin(\phi),
\end{aligned} \tag{3.3}$$

and these coordinates cover H^4 globally if we let Ψ run from $-\infty$ to $+\infty$ while Σ runs from 0 to $+\infty$. [Both θ and ϕ are suppressed in Figure 1.]

JHEP10(2006)029

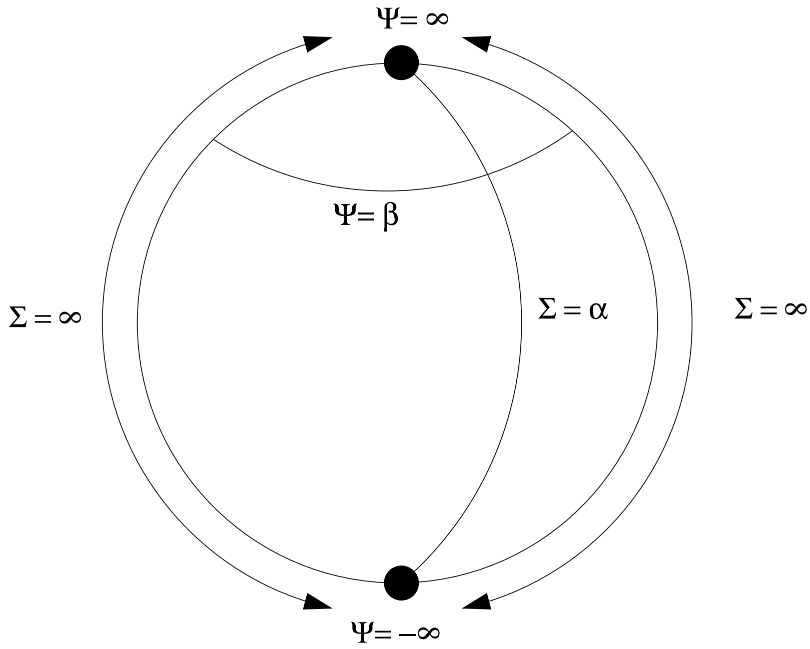

Figure 1: Zero extrinsic curvature foliation of H^4.

The surfaces $\Sigma =$ constant are topological cylinders; recall that a cylinder, with topology $\mathbb{R} \times S^2$, is a three-sphere from which two points have been deleted. If we think of H^4 as the interior of a four-dimensional ball, then these cylinders are "pinched together" as they approach the boundary at the points $\Psi = \pm\infty$. A typical "pinched cylinder" inside the boundary is shown[4] as $\Sigma = \alpha =$ constant in Figure 1. It is clear that the conformal boundary is a cylinder with two additional points [corresponding to $\Psi = \pm\infty$] added: with these additions, the boundary becomes the familiar three-sphere. Notice that the structure of the interior, partitioned into these cylinders, is very similar to that of the simply connected version of Lorentzian anti-de Sitter spacetime [15][16]. Notice too that the cylinders themselves *do not* intersect — they only do so in the conformal completion.

The foliation in which we are interested here is given by the surfaces $\Psi =$ constant, transverse to the cylinders. These are copies of three-dimensional hyperbolic space; all of them have *the same intrinsic curvature*, $-1/L^2$, as each other and as the ambient H^4. This can be seen by noting that the first two equations of (3.3) imply that $-A^2 + B^2$ is independent of Ψ. All of these slices intersect the conformal boundary at right angles, and all of them have *zero extrinsic curvature.* A typical surface $\Psi = \beta =$ constant is shown in Figure 1.

Because the slices have zero extrinsic curvature, Ψ can be compactified: the consequences for Figure 1 are shown in Figure 2: parts of the diagram have to be deleted, and the top and bottom of the diagram have to be identified. The boundary changes topology from S^3 to $S^1 \times S^2$. This is one possible form of "hyperbolic space" if, as is frequently the

[4]In dimensions above two, the reader can picture such a cylinder by rotating the $\Sigma = \alpha$ line about an axis passing through the heavy dots.

JHEP10(2006)029

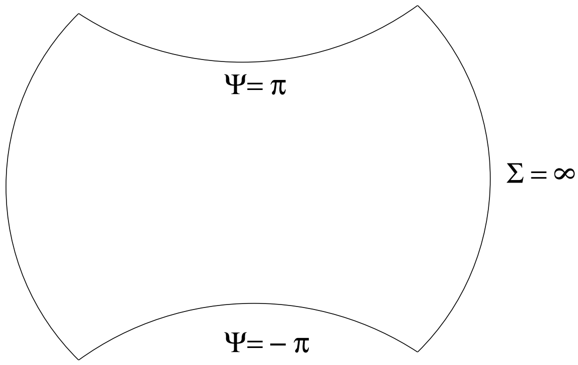

Figure 2: ZEC foliation of H^4, with compactification of Ψ axis.

case, one wants Euclidean time to be periodic.

The metric [with constant curvature $-1/L^2$] expressed in terms of the coordinates defined by this Zero Extrinsic Curvature foliation is

$$g(H^4; \mathrm{ZEC})_{++++} \; = \; L^2 \left\{ \cosh^2(\Sigma)\, d\Psi^2 \; + \; d\Sigma^2 \; + \; \sinh^2(\Sigma)[d\theta^2 \; + \; \sin^2(\theta) d\phi^2] \right\}. \quad (3.4)$$

We now complexify $\Psi \rightarrow iU/L$ [keeping U/L periodic if Ψ is compactified as in Figure 2] and re-label Σ [without complexifying it] as S/L, then we obtain, from (3.4),

$$g(\mathrm{AdS}_4)_{-+++} \; = \; -\cosh^2(S/L)\, dU^2 \; + \; dS^2 \; + \; L^2 \sinh^2(S/L)[d\theta^2 \; + \; \sin^2(\theta) d\phi^2], \quad (3.5)$$

and this is precisely [14] the *globally valid* AdS_4 metric, in the indicated signature. The basic definition of AdS, as a locus in a higher-dimensional space, leads to cyclic time[5], so with angular U/L this is indeed precisely AdS_4; while we are free to take the universal cover, it can be argued [15][16] that this is not really necessary. In other words, Figure 2 is relevant in the case where either Euclidean or Lorentzian time is periodic.

This calculation provides a rigorous basis for the standard claim that anti-de Sitter spacetime is the complexified version of hyperbolic space. It is clear, however, that we obtained this result by *choosing* a very specific foliation of H^4 — one with leaves having zero extrinsic curvature. By doing this, we guarantee that the complexified version will have a timelike Killing vector; but, again, this is a matter of deliberate construction, not something that is forced on us. [Notice in this connection that if we complexify Σ instead of Ψ in (3.4), then the result is the *static* version of the local de Sitter metric, with $(+ - - -)$ signature.] If, following Ooguri et al, we declare that other foliations of H^4 are acceptable, then we can expect to obtain a dynamical spacetime upon complexification. Let us see how this works.

3.2 Foliation corresponding to spatially spherical de Sitter

Now we shall consider a second, completely different, but also entirely global foliation of H^4, shown in Figure 3. Choose coordinates Θ,ρ,θ,ϕ, where Θ runs from $-\infty$ to $+\infty$ while

[5] Note that the hyperbolic functions of Ψ in the relations (3.3) become trigonometric, therefore periodic, when Ψ is complexified.

JHEP10(2006)029

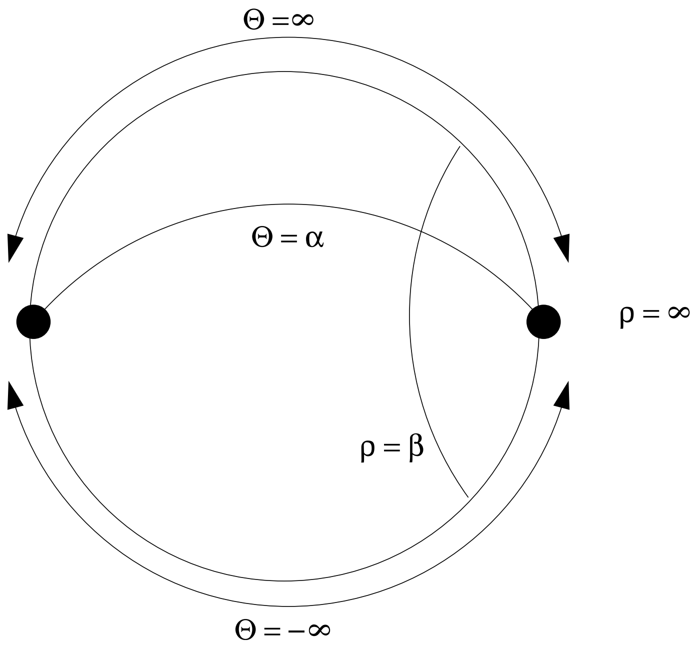

Figure 3: Θ foliation of H^4.

ρ runs from 0 to $+\infty$, and set

$$\begin{aligned} A &= L\cosh(\Theta)\cosh(\rho) \\ B &= L\sinh(\Theta) \\ Z &= L\cosh(\Theta)\sinh(\rho)\cos(\theta) \\ Y &= L\cosh(\Theta)\sinh(\rho)\sin(\theta)\cos(\phi) \\ X &= L\cosh(\Theta)\sinh(\rho)\sin(\theta)\sin(\phi). \end{aligned} \tag{3.6}$$

Then the leaves of the foliation are labelled by Θ. We shall call this the "Θ foliation".

Because we are suppressing two angles, Figure 3 seems to resemble Figure 1, but this is misleading [except in two dimensions, see below]. Here the surfaces $\Theta =$ constant are, from the second member of equations (3.6), just the submanifolds $B =$ constant; they are copies of the three-dimensional hyperbolic space H^3, as can be seen at once from equation (3.1). This foliation differs from the previous one in a crucial way, however: whereas previously the slices all had the same intrinsic curvature, $-1/L^2$, as the ambient space, here the surface $\Theta = \alpha =$ constant can be written as

$$-A^2 + X^2 + Y^2 + Z^2 = -L^2\cosh^2(\alpha), \tag{3.7}$$

so the magnitude of the intrinsic curvature of a slice is reduced by a factor of $\mathrm{sech}^2(\alpha)$. The slices become flatter as they expand towards the boundary. Their extrinsic curvature is therefore *never zero*, with the sole exception of the equatorial slice at $\Theta = 0$. In this case, the copies of H^3 are all "pinched together" as we move towards *their* boundaries, that is, as $\rho \to \infty$. A typical H^3 slice, $\Theta = \alpha =$ constant is shown in Figure 3.

JHEP10(2006)029

Notice that the slices themselves do not intersect: only their conformal completions do so. At any point actually in a given copy of H^3 [and not on its conformal boundary], one can send a geodesic [shown in Figure 3] of the form $\theta = \phi =$ constant, $\rho =$ constant $= \beta$, towards infinity, and this will uniquely define two points on the boundary, one each at $\Theta = \pm\infty$. From this point of view, one can say that the conformal infinity of H^4 is "finitely disconnected": the usual boundary three-sphere is divided into two hemispheres corresponding to the forward or backward "evolution" along the geodesics perpendicular to these slices. Of course, topologically the boundary is connected, since the two hemispheres join along the common conformal boundary of all of the slices; but this can only be detected by proceeding to infinity in the "spacelike" direction, that is, along the slices.

It is clear that this foliation, like the previous one, foliates H^4 globally, though it is in general totally different to the one shown in Figure 1. The metric [with curvature $-1/L^2$] with respect to this foliation is

$$g(H^4;\Theta)_{++++} \;=\; L^2\left\{d\Theta^2 \;+\; \cosh^2(\Theta)[d\rho^2 \;+\; \sinh^2(\rho)\{d\theta^2 \;+\; \sin^2(\theta)\,d\phi^2\}]\right\}. \quad (3.8)$$

If we now complexify by mapping $\rho \rightarrow \pm i\chi$ while re-labelling Θ as T/L, we obtain, since $\sinh(\pm\, i\,\chi) = \pm i\sin(\chi)$,

$$g(SSdS_4)_{+---} \;=\; dT^2 \;-\; L^2\cosh^2(T/L)\,[d\chi^2 \;+\; \sin^2(\chi)\{d\theta^2 \;+\; \sin^2(\theta)\,d\phi^2\}], \quad (3.9)$$

which is of course the global, Spatially Spherical form of the de Sitter metric, but now in $(+\;-\;-\;-)$ signature. We have become accustomed to thinking of de Sitter spacetime as a space of *positive* curvature, but we again remind the reader that this is a matter of convention [of the signature]: in $(+\;-\;-\;-)$ signature, de Sitter spacetime has *negative* curvature, and there is no sense in which this is less natural than positive curvature in the opposite convention.

Thus de Sitter and anti-de Sitter spacetimes are seen to have a common origin in *different foliations* of the *same* Euclidean (hyperbolic) space[6]. In this case — unlike that of the sphere considered earlier — *both* complexifications lead to the full, global Lorentzian versions of the spacetimes in question.

Before proceeding, let us settle a technical point. One can regard analytic continuation as a mere technical device, a solution-generating technique. But if one wishes to use it in the original way, to construct the Euclidean gravity path integral [21], then it is important that the procedure should also complexify the volume form. In equation (3.8), for example, the volume form is

$$dV(g(H^4;\Theta)_{++++}) \;=\; L^4\cosh^3(\Theta)\sinh^2(\rho)\sin(\theta)\,d\Theta\,d\rho\,d\theta\,d\phi, \quad (3.10)$$

and one sees at once that complexifying ρ [to obtain equation (3.9)] does indeed complexify the volume form. However, this only works if the number of spacetime dimensions is *even*. Thus we shall confine ourselves to even spacetime dimensions henceforth. [Depending on

[6]The idea that distinct foliations can have distinct physics was proposed in [19][20].

JHEP10(2006)029

the dimension, one may have to choose the sign of the imaginary factor in the complexification so that the volume form "rotates" in the correct direction. This is the reason for the $\pm$ sign in the complexification of ρ, above.]

The observation that de Sitter spacetime, like anti-de Sitter spacetime, has a natural association with hyperbolic space was suggested in [22]; related ideas were investigated in [23]; it has been put on a rigorous mathematical basis [though mainly in the case of Einstein bulks, which are of limited cosmological interest] by Anderson [24]; and it is relevant to any theory which makes use of the fact that the de Sitter and anti-de Sitter spacetimes are mutually locally conformal [25][26].

The fact that hyperbolic space can be complexified to de Sitter spacetime gives reason to hope that it should indeed be possible to realise the suggestion of Ooguri et al that the metric in equation (1.1) has a cosmological interpretation. With this in mind, we proceed to yet another foliation of hyperbolic space.

3.3 Foliation corresponding to spatially hyperbolic de Sitter

Since $\mathrm{A} \geq \mathrm{L}$ in equation (3.1), the most obvious way to choose coordinates here is to define $\mathrm{P},\chi,\theta,\phi$ such that

$$\begin{aligned} \mathrm{A} &= \mathrm{L}\cosh(\mathrm{P}/\mathrm{L}) \\ \mathrm{B} &= \mathrm{L}\sinh(\mathrm{P}/\mathrm{L})\cos(\chi) \\ \mathrm{Z} &= \mathrm{L}\sinh(\mathrm{P}/\mathrm{L})\sin(\chi)\cos(\theta) \\ \mathrm{Y} &= \mathrm{L}\sinh(\mathrm{P}/\mathrm{L})\sin(\chi)\sin(\theta)\cos(\phi) \\ \mathrm{X} &= \mathrm{L}\sinh(\mathrm{P}/\mathrm{L})\sin(\chi)\sin(\theta)\sin(\phi), \end{aligned} \tag{3.11}$$

giving the familiar Poincaré "disc" representation of hyperbolic space, with Poincaré radial coordinate P and metric

$$g(\mathrm{H}^4;\mathrm{P})_{++++} \;=\; +\,\mathrm{dP}^2 \;+\; \mathrm{L}^2\sinh^2(\mathrm{P}/\mathrm{L})\,[\mathrm{d}\chi^2 \;+\; \sin^2(\chi)\{\mathrm{d}\theta^2 \;+\; \sin^2(\theta)\,\mathrm{d}\phi^2\}]. \tag{3.12}$$

We are now foliating H^4 by spheres labelled by P, which as usual is "Euclidean time". In many ways this is the most natural way to picture hyperbolic space — for example, it makes the [spherical] structure at infinity very clear. However, this version of H^4 does not usually appear in discussions of the Euclidean form of the AdS/CFT correspondence, for the simple reason that the usual $(+\;+\;+\;+)\to(-\;+\;+\;+)$ continuation is not possible here: in this case, *only* the $(+\;+\;+\;+)\to(+\;-\;-\;-)$ continuation actually works[7]. For if we attempt to complexify P, the result is not Lorentzian; while if we complexify χ and re-label suitably we obtain

$$g(\mathrm{SHdS}_4)_{+---} \;=\; +\,\mathrm{d}\tau^2 \;-\; \sinh^2(\tau/\mathrm{L})\,[\mathrm{dr}^2 \;+\; \mathrm{L}^2\sinh^2(\mathrm{r}/\mathrm{L})\{\mathrm{d}\theta^2 \;+\; \sin^2(\theta)\,\mathrm{d}\phi^2\}]. \tag{3.13}$$

This is actually yet another version of de Sitter spacetime, one with *negatively* curved spatial sections instead of local spheres. This is the prototype for "hyperbolic" accelerating cosmologies, which have recently attracted much attention from various points of

[7] By contrast, we saw that the metric in (3.4) has a second continuation to the static patch of de Sitter, and similarly the metric in (3.8) can be continued also to Dynamical AdS.

JHEP10(2006)029

view [27][28][29][30][31]. This spacetime is therefore potentially of very considerable interest.

As is explained in [32], the coordinates in equation (3.13) only cover the interior of the future lightcone of a point on the equator of global de Sitter spacetime. In this sense, this metric represents not a spacetime, but rather a *part* of a spacetime; this part is geodesically incomplete in a way that is physically meaningless and that would forbid "creation from nothing". As in our discussion of Dynamical AdS in Section 2 above, we take this as an instruction to compactify the spatial sections: we interpret the three-dimensional metric on the spatial sections in equation (3.13) as a metric on a compact space of the form H^3/Γ, where Γ is some discrete freely acting infinite group of H^3 isometries such that the quotient is compact. One can think of this in the following way: we are effectively imposing certain [very intricate] restrictions on the ranges of the coordinates in the spatial part of the metric[8]. Note that in general Γ will be a very complicated object, but also that this complexity may have a direct physical meaning in connection with the way string theory may possibly resolve the "singularity" at $\tau = 0$; see [29][31] for the details. In this work, however, the geometry around $\tau = 0$ will be regularized in a different and more direct way, since in any case we will truncate the spacetime at some non-zero value of τ, in the usual manner of "creation from nothing".

Compactifying the spatial slices produces a physically distinct spacetime. It is no longer possible for objects to enter the spacetime from "outside", except at $\tau = 0$. That is, the spacetime is still geodesically incomplete, but only at $\tau = 0$; this is reasonable physically, since the introduction of conventional matter into this spacetime can in any case be expected to generate a curvature singularity at that point[9]. More relevantly here, we are interested in creating these universes from "nothing", and, in that context, the spacetime would in any case be truncated at some value of τ strictly greater than zero, where there would be a transition from a Euclidean metric to a Lorentzian one; so the incompleteness at $\tau = 0$ is physically irrelevant. Thus the problem of objects entering or leaving the spacetime along some null surface has been solved.

To summarize: the Poincaré "disc" model of H^4 can be complexified, but *only* in the $(+\ +\ +\ +)\rightarrow(+\ -\ -\ -)$ sense. The result is the very interesting Spatially Hyperbolic version of de Sitter spacetime.

Finally, we turn to the foliation of hyperbolic space which is actually the one used by Ooguri et al [1].

3.4 Foliation corresponding to spatially flat de Sitter

We define coordinates Φ, x, y, z on H^4 by

$$\mathrm{A} = \mathrm{L}\cosh(\Phi/\mathrm{L}) \;+\; \frac{1}{2\mathrm{L}}\,(\mathrm{x}^2 \;+\; \mathrm{y}^2 \;+\; \mathrm{z}^2)\,\mathrm{e}^{-\,\Phi/\mathrm{L}}$$

[8] See [33] for a discussion of this in the much simpler case of Spatially Toral de Sitter spacetime, STdS.

[9] Since the Strong Energy Condition does not hold here, one cannot prove this using the classical singularity theorems; instead one invokes the recent results of Andersson and Galloway [34]; see also [35] and [33] for a discussion.

JHEP10(2006)029

$$
\begin{aligned}
\mathrm{B} &= \mathrm{L}\sinh(\Phi/\mathrm{L}) \;+\; \frac{1}{2\mathrm{L}}\,(\mathrm{x}^2 \;+\; \mathrm{y}^2 \;+\; \mathrm{z}^2)\,\mathrm{e}^{-\,\Phi/\mathrm{L}} \\
\mathrm{Z} &= \mathrm{z}\,\mathrm{e}^{-\,\Phi/\mathrm{L}} \\
\mathrm{Y} &= \mathrm{y}\,\mathrm{e}^{-\,\Phi/\mathrm{L}} \\
\mathrm{X} &= \mathrm{x}\,\mathrm{e}^{-\,\Phi/\mathrm{L}}.
\end{aligned} \tag{3.14}
$$

Here all coordinates run from $-\infty$ to $+\infty$, and the metric is

$$
g(\mathrm{H}^4;\Phi)_{++++} \;=\; \mathrm{d}\Phi^2 \;+\; \mathrm{e}^{(-\,2\,\Phi/\mathrm{L})}\,[\,\mathrm{dx}^2 \;+\; \mathrm{dy}^2 \;+\; \mathrm{dz}^2]. \tag{3.15}
$$

Evidently the surfaces Φ = constant are infinite, flat spaces of topology $\mathbb{R}^3$. In order to understand how these fit into the Poincaré disc, recall that, by stereographic projection, $\mathbb{R}^3$ has the same topology as a three-sphere from which one point has been deleted. If, therefore, we take a finite sphere in the Poincaré disc and move it until it touches the boundary sphere at one point [which we take to be the "north pole" of the disc coordinates, $\chi = 0$ in equations (3.11)], the part of the sphere which lies in the bulk is in fact a copy of $\mathbb{R}^3$ [see Figure 4]. A collection of such copies of $\mathbb{R}^3$, all obtained from spheres intersecting at the same point on the conformal boundary, foliate the entire bulk of H^4; see Figure 4. The metric corresponding to this foliation is precisely the one given in (3.15). The value of Φ corresponds to the size of the sphere in the figure: larger spheres correspond to negative values of Φ, which runs from bottom to top in the figure.

As in the cases with hyperbolic spatial sections, the Lorentzian version of this geometry will be geodesically incomplete along a null surface, ruling out "creation from nothing", unless we compactify the $\mathbb{R}^3$ slices. They can of course be compactified to topology $\mathbb{R}^3/\mathbb{Z}^3$ [among other possibilities] by simply setting $\mathrm{x} = \mathrm{K}\theta_1$, $\mathrm{y} = \mathrm{K}\theta_2$, $\mathrm{z} = \mathrm{K}\theta_3$, for some positive constant K, where $\theta_{1,2,3}$ are angular coordinates. The slices are now cubic tori, with a size which "evolves" from infinity at $\Phi = -\infty$ to zero at $\Phi = +\infty$. The metric is now

$$
g(\mathrm{H}^4/\mathbb{Z}^3;\Phi)_{++++} \;=\; \mathrm{d}\Phi^2 \;+\; \mathrm{K}^2\,\mathrm{e}^{(-\,2\,\Phi/\mathrm{L})}\,[\,\mathrm{d}\theta_1^2 \;+\; \mathrm{d}\theta_2^2 \;+\; \mathrm{d}\theta_3^2]. \tag{3.16}
$$

This is just the four-dimensional version of the metric (1.1) considered by Ooguri et al [1]. The manifold can be approximately portrayed as the region between the lines extending down from the north pole in Figure 4: the idea is that one keeps only a finite piece of each $\mathbb{R}^3$ section of the full space, and then performs the required identifications on this piece.

This space has two complexifications. One can of course complexify θ_1, thereby obtaining ["Partly Compactified"] anti-de Sitter spacetime:

$$
g(\mathrm{PCAdS}_4)_{-+++} \;=\; \mathrm{d}\Phi^2 \;+\; \mathrm{K}^2\,\mathrm{e}^{(-\,2\,\Phi/\mathrm{L})}\,[-\,\mathrm{d}\theta_1^2 \;+\; \mathrm{d}\theta_2^2 \;+\; \mathrm{d}\theta_3^2]. \tag{3.17}
$$

But there is another, less obvious complexification.

Ooguri et al observe that one cannot complexify Φ here, so that one cannot obtain a complexification of the $(++++)\rightarrow(-+++)$ kind in that way. Instead we proceed in the now familiar manner: we search for a complexification of the form $(++++)\rightarrow(+---)$. This can be achieved here in a particularly elegant manner if we define a dimensionless [angular] Euclidean "conformal time" η_-, taking values in the range $(-\infty, 0)$, by

$$
\eta_- \;=\; -\,\pi\,\mathrm{e}^{\Phi/\mathrm{L}}. \tag{3.18}
$$

JHEP10(2006)029

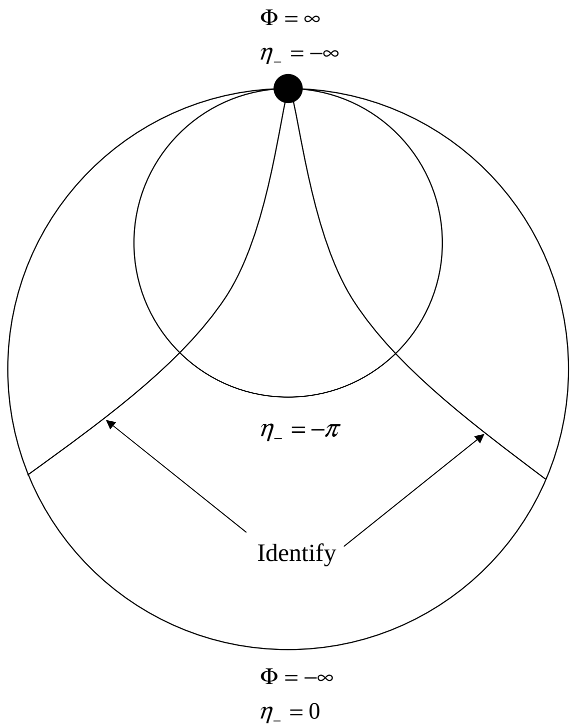

Figure 4: $\mathbb{R}^3$ foliation of H^4, and its partial compactification.

JHEP10(2006)029

We now have

$$g(\mathrm{H}^4/\mathbb{Z}^3;\eta_-)_{++++} \;=\; \frac{1}{\eta_-^2}\,[\mathrm{L}^2\mathrm{d}\eta_-^2 \;+\; \pi^2\,\mathrm{K}^2\,\{\mathrm{d}\theta_1^2 \;+\; \mathrm{d}\theta_2^2 \;+\; \mathrm{d}\theta_3^2\}]. \tag{3.19}$$

We now complexify $\eta_- \;\to\; \pm\mathrm{i}\eta_+$, where η_+ takes its values in $(0,\,\infty)$. The result is the well-known spatially flat version of Lorentzian de Sitter spacetime, but now with toral sections: it is Spatially Toral de Sitter, in $(+\;-\;-\;-)$ signature:

$$\begin{aligned} g(\mathrm{STdS}_4)_{+---} &= \frac{1}{\eta_+^2}\,[\mathrm{L}^2\mathrm{d}\eta_+^2 \;-\; \pi^2\,\mathrm{K}^2\,\{\mathrm{d}\theta_1^2 \;+\; \mathrm{d}\theta_2^2 \;+\; \mathrm{d}\theta_3^2\}] \\ &= \mathrm{dt}^2 \;-\; \mathrm{K}^2\,\mathrm{e}^{(2\,\mathrm{t}/\mathrm{L})}\,[\mathrm{d}\theta_1^2 \;+\; \mathrm{d}\theta_2^2 \;+\; \mathrm{d}\theta_3^2], \end{aligned} \tag{3.20}$$

where t, which ranges from $-\infty$ to $+\infty$, is related to η_+ by

$$\eta_+ \;=\; +\pi\, \mathrm{e}^{-\,\mathrm{t/L}}. \tag{3.21}$$

We stress again that this spacetime is topologically and physically *distinct* from both the Spatially Spherical and the Spatially Hyperbolic de Sitter spacetimes.

Thus we have succeeded in associating an accelerating Lorentzian cosmology with the partially compactified hyperbolic space $\mathrm{H}^4/\mathbb{Z}^3$, just as Ooguri et al require.

In Section 2 we saw that there were two distinct complexifications of the sphere, arising from two distinct but equally valid ways of foliating it. We also saw, however, that one complexification was favoured over the other, because one of the resulting Lorentzian spacetimes failed to attain macroscopic size: it collapsed immediately after being created. In the hyperbolic case, by contrast, nothing of this sort happens: one complexification leads to AdS_4, while the other three lead to three physically distinct families[10] of expanding, accelerating spacetimes. How can this ambiguity be resolved?

In order to answer this, let us consider the peculiarities of the two-dimensional case, since that is the case discussed in [1].

4. The two-dimensional case and what it teaches us

In the two-dimensional case, Figures 1, 2, 3, and 4 can be interpreted literally, in the sense that there are no angles to be suppressed. In particular, a simple reflection, $\Psi \to \rho$, $\Sigma \to \Theta$ shows that the two foliations in Figures 1 and 2 are identical. More interesting is the fact that the corresponding Lorentzian spaces are also identical: in the two-dimensional case we have [from equations (3.5) and (3.9)]

$$g(\mathrm{AdS}_2)_{-+} \;=\; -\cosh^2(\mathrm{S/L})\,\mathrm{dU}^2 \;+\; \mathrm{dS}^2, \tag{4.1}$$

$$g(\mathrm{SSdS}_2)_{+-} \;=\; \mathrm{dT}^2 \;-\; \mathrm{L}^2\cosh^2(\mathrm{T/L})\,\mathrm{d}\chi^2, \tag{4.2}$$

where, as before, we regard both U/L and χ as angular coordinates. This corresponds to the fact that the anti-de Sitter group in n+1 dimensions, O(2,n), is isomorphic to the de Sitter group O(n+1,1), when n = 1. [Actually the symmetry groups are smaller, since we compactify U and χ, but these smaller groups are the same for both spacetimes]. It is helpful to consider how "Euclidean time" works in Figure 3. In the "AdS" case one thinks of "time" as running vertically [so that the boundary is at "spatial" infinity], while in the "dS" case one reflects the diagram about a diagonal so that the boundary is in the "future" and "past". Of course, neither definition of Euclidean "time" is more valid than the other, and this statement is the basis of the dual interpretation of two-dimensional hyperbolic space introduced by Ooguri et al.

In view of this, the claim of Ooguri et al, that one can create from "nothing" an accelerating two-dimensional cosmology using [a version of] *negatively* curved hyperbolic

[10] "Families", because in each case there are many possible compactifications of the spatial sections: see [36][37][29].

JHEP10(2006)029

space, clearly must be valid. All that remains is to see how the derivation works in a technical sense. We claim that the necessary technical device is precisely the very mild generalization of complexification that we have introduced here. Let us see how this works in detail.

First, take equation (4.2) and split the spacetime along its spacelike hypersurface of zero extrinsic curvature at $\mathrm{T} = 0$, retaining only the $\mathrm{T} \geq 0$ half. Similarly we can take the two-dimensional version of equation (3.8), compactify the coordinate ρ [so that we are dealing with $\mathrm{H}^2/\mathbb{Z}$ rather than H^2 itself], and obtain

$$g(\mathrm{H}^2/\mathbb{Z};\Theta)_{++} \;=\; \mathrm{L}^2\,[\mathrm{d}\Theta^2 \;+\; \cosh^2(\Theta)\mathrm{d}\rho^2]; \tag{4.3}$$

this too splits naturally at $\Theta = 0$, and we retain only $\Theta \;\leq\; 0$. Both $\mathrm{T} = 0$ and $\Theta = 0$ are surfaces of zero extrinsic curvature, and both are circles of radius L. We combine the two halves along these surfaces, obtaining a manifold of topology $\mathbb{R} \times \mathrm{S}^1$ with a metric which we symbolize by

$$g(\mathrm{H}^2/\mathbb{Z};\Theta \;\leq\; 0)_{++} \;\longrightarrow\; g(\mathrm{SSdS}_2;\;\mathrm{T} \;\geq\; 0)_{+-}. \tag{4.4}$$

The arrow here represents the idea that the Euclidean version is succeeded by the Lorentzian version at the creation.) Of course, this just means that we have a Euclidean-to-Lorentzian transition, with the first metric valid on one side, the second on the other. This would be the geometry underlying the creation of this version of two-dimensional de Sitter spacetime, as described by the Ooguri et al negatively-curved analogue of the hemisphere used to construct the Hartle-Hawking wave function.

We can repeat this for the other two versions of two-dimensional de Sitter: from equation (3.13) we have

$$g(\mathrm{SHdS}_2)_{+-} \;=\; +\,\mathrm{d}\tau^2 \;-\; \sinh^2(\tau/\mathrm{L})\,\mathrm{dr}^2, \tag{4.5}$$

where our agreed compactification of the spatial sections means that r is proportional to some angular coordinate, so that the τ = constant sections are circular. This is to be compared with the two-dimensional version of (3.12),

$$g(\mathrm{H}^2;\mathrm{P})_{++} \;=\; +\,\mathrm{dP}^2 \;+\; \mathrm{L}^2\sinh^2(\mathrm{P}/\mathrm{L})\,\mathrm{d}\chi^2; \tag{4.6}$$

here it will be useful to define P as having negative values ranging from $-\infty$ to zero.

Notice that there is an important difference between this case and the previous one: here there is no surface of zero extrinsic curvature in either the Lorentzian or the Euclidean cases. As this is also a property of the spatially flat case considered by Ooguri et al [see below], we postpone discussion of this point; for the moment let us arbitrarily truncate the range of τ to $[\alpha,\,\infty)$ for some positive constant α with dimensions of length, and that of P to $(-\infty,\,-\alpha]$, so that at both $\tau = \alpha$ and $\mathrm{P} = -\alpha$ we have circular sections of radius $\mathrm{L}\sinh(\alpha/\mathrm{L})$. Joining the two spaces along these circles, we obtain a space of topology $\mathbb{R} \times \mathrm{S}^1$ with a metric

$$g(\mathrm{H}^2;\mathrm{P} \;\leq\; -\,\alpha)_{++} \;\longrightarrow\; g(\mathrm{SHdS}_2;\;\tau \;\geq\; \alpha)_{+-}. \tag{4.7}$$

JHEP10(2006)029

Assuming that the truncations can be justified, this would be the geometry underlying the description by the OVV wave function of the creation of Spatially Hyperbolic de Sitter spacetime from "nothing".

Finally we come to the case actually studied by Ooguri et al, with flat, toral spatial sections. The Euclidean metric in this case is just the two-dimensional version of (3.16) and (3.19),

$$\begin{aligned} g(\mathrm{H}^2/\mathbb{Z};\eta_-;\Phi)_{++} \;&=\; \frac{1}{\eta_-^2}\,[\mathrm{L}^2\,\mathrm{d}\eta_-^2 \;+\; \pi^2\,\mathrm{K}^2\,\mathrm{d}\theta_1^2] \\ &=\; \mathrm{d}\Phi^2 \;+\; \mathrm{K}^2\,\mathrm{e}^{(-\,2\,\Phi/\mathrm{L})}\,\mathrm{d}\theta_1^2, \end{aligned} \tag{4.8}$$

and of course we wish to combine this with the two-dimensional version of (3.20),

$$\begin{aligned} g(\mathrm{STdS}_2)_{+-} &= \frac{1}{\eta_+^2}\,[\mathrm{L}^2\,\mathrm{d}\eta_+^2 \;-\; \pi^2\,\mathrm{K}^2\,\mathrm{d}\theta_1^2] \\ &= \mathrm{d}t^2 \;-\; \mathrm{K}^2\,\mathrm{e}^{(2\,t/\mathrm{L})}\,\mathrm{d}\theta_1^2. \end{aligned} \tag{4.9}$$

As in the case of hyperbolic sections, the absence of any surface of zero extrinsic curvature here means that we have to truncate the ranges of η_+ and η_-. Since the other coordinate, θ_1, is angular [ranging from $-\,\pi$ to $+\,\pi$], it is natural to truncate η_- at $\eta_- = -\,\pi$, so that the range of this coordinate is $[-\,\pi,\,0)$ — see Figure 4. This truncates the space along a circle which [by equation (4.8)] is of circumference $2\pi\mathrm{K}$. To ensure continuity, the Lorentzian spacetime must also be truncated along a circle of circumference $2\pi\mathrm{K}$. This, by equation (4.9), means that the range of Lorentzian conformal time is $(0,\,\pi]$. We then topologically identify the circle at $\eta_- = -\,\pi$ with the circle at $\eta_+ = \pi$; *thus, K is the initial radius of the Universe at the moment of creation.*

The fact that we are taking the range of all "angular" coordinates to be from $-\,\pi$ to $+\,\pi$ now neatly reflects the fact that the Euclidean and Lorentzian spaces are identified along their edges. Note that this angular interpretation of Euclidean and Lorentzian conformal time suggests that we should consider a topological identification of the Euclidean conformal boundary [$\eta_- = 0$] with the Lorentzian future spacelike conformal infinity [$\eta_+ = 0$]. The full conformal compactification will then itself be a [two-dimensional] torus. Roughly speaking, L is the radius of this torus in one direction, while K is its radius in the other. We shall return to this in the Conclusion.

We now define a metric on $\mathbb{R} \times \mathrm{S}^1$ by

$$g(\mathrm{H}^2/\mathbb{Z};\; -\,\pi \,\leq\, \eta_- \,<\, 0)_{++} \;\longrightarrow\; g(\mathrm{STdS}_2;\; 0 \,<\, \eta_+ \,\leq\, \pi)_{+-}. \tag{4.10}$$

This is the geometry describing the creation of an accelerating Universe, with flat [but compact] spatial sections, in the OVV picture.

The fact that one has to truncate both the spatially hyperbolic and the spatially toral versions of de Sitter spacetime is due to the structure of the Einstein equations [in the form of the Friedmann equations], which forbid the extrinsic curvature of any hypersurface to vanish. This means that the Euclidean-to-Lorentzian transition can be continuous but not smooth. Of course, one has every reason to suspect that the Einstein equations do not

JHEP10(2006)029

hold exactly at the transition point, and that the corrected equations will allow a smooth transition. This proves to be so, and we shall discuss the details [for toral sections] when we return to the four-dimensional case in the next section.

The two-dimensional case is interesting partly because it arises naturally in the context considered by Ooguri et al, and partly because it teaches us that there must be some natural way of associating an accelerating Lorentzian universe with a negatively curved Euclidean space. We have argued that there is indeed a very simple way of establishing such an association: complexify according to [the two-dimensional version of] $(+\ +\ +\ +)\rightarrow(+\ -\ -\ -)$ instead of $(+\ +\ +\ +)\rightarrow(-\ +\ +\ +)$. If we proceed in this way, we find that we can set up the geometric background for the OVV version of creation from "nothing": indeed, we can do this for all three versions of de Sitter spacetime. This last point is somewhat disappointing, since one might have hoped that the OVV wave function might give us a clue as to which version is the correct one. As we shall now see, the situation in four dimensions is much more satisfactory in this regard.

5. Four dimensions

In four dimensions, we are again interested in three metrics of constant negative curvature: $g(\mathrm{H}^4;\Theta)_{++++}$ [equation (3.8)], $g(\mathrm{H}^4;\mathrm{P})_{++++}$ [equation (3.12)], and $g(\mathrm{H}^4/\mathbb{Z}^3)_{++++}$ [equation (3.19)].

But now we find something remarkable: if we try to generalize the discussion of the preceding section to the four-dimensional case, it *does not work* for $g(\mathrm{H}^4;\Theta)_{++++}$ and $g(\mathrm{H}^4;\mathrm{P})_{++++}$. For the transverse sections defined by $g(\mathrm{H}^4;\Theta)_{++++}$ are negatively curved: they cannot be joined to the spacelike sections of the Lorentzian version, $g(\mathrm{SSdS}_4)_{+---}$ [equation (3.9)], since these are positively curved. Similarly, the transverse sections defined by $g(\mathrm{H}^4;\mathrm{P})_{++++}$ are positively curved, and cannot be joined to the negatively curved spacelike sections of $g(\mathrm{SHdS}_4)_{+---}$ [equation (3.13)].

This difficulty did not arise in the two-dimensional case, for the simple reason that the spatial sections of a two-dimensional spacetime are one-dimensional, and of course one-dimensional manifolds cannot be curved either positively or negatively. The only case where this is not a problem in higher dimensions is the case where the spatial sections are *flat*, since a reversal of the sign of the curvature has no effect here. This is of course precisely the case considered by Ooguri et al.

Hartle [12] has recently argued that Lorentzian signature "emerges" from the formalism of Euclidean quantum gravity. If we begin with a spherical Euclidean geometry, the geometry cannot remain Euclidean if its sections are to become significantly larger than the curvature scale: there has to be a transition to a different signature for this to be possible. But this signature cannot be to $(+\ +\ -\ -)$ or $(-\ -\ +\ +)$, since the three-dimensional sections in such a case would themselves be Lorentzian, not Euclidean. Thus Hartle claims that Lorentzian signature is "emergent" within this theory of quantum gravity. In precisely the same way, we find that *the toral structure of the spatial sections of our Universe is emergent within the theory of Ooguri et al:* foliations of the Euclidean space with either

JHEP10(2006)029

positively or negatively curved sections are unable to make the transition to the Lorentzian regime[11].

In the toral case, and in this case *only*, we can generalize the discussion of the preceding section: we truncate $\mathrm{H}^4/\mathbb{Z}^3$ so that η_- takes values in $[-\pi,\, 0)$, while Spatially Toral de Sitter, STdS_4, is truncated so that η_+ takes values in $(0,\, \pi]$; both spaces are joined along a three-dimensional torus consisting of circles of circumference $2\pi K$. Then we can define a metric on $\mathbb{R} \times \mathrm{T}^3$ by

$$g(\mathrm{H}^4/\mathbb{Z}^3;\; -\pi \,\leq\, \eta_- \,<\, 0)_{++++} \;\longrightarrow\; g(\mathrm{STdS}_4;\; 0 \,<\, \eta_+ \,\leq\, \pi)_{+---}, \tag{5.1}$$

where the change of signature is effected by complexifying *conformal* time. [Recall that the arrow symbolizes the transition from a Euclidean to a Lorentzian metric, and bear in mind that the Euclidean-to-Lorentzian transition occurs along $\eta_- = -\pi$ and $\eta_+ = \pi$, not at $\eta_\pm = 0$.] This should describe the creation of the universe [at $\eta_- = -\pi$ in Figure 4] in terms of a four-dimensional version of the Ooguri et al wave function, defined on $\mathrm{H}^4/\mathbb{Z}^3$. Again, it may be of interest to consider identifying the Euclidean infinity at $\eta_- = 0$ with Lorentzian future spacelike infinity at $\eta_+ = 0$, as suggested by the angular interpretation of η_+ and η_-.

As in the two-dimensional case, the Euclidean-to-Lorentzian transition here is continuous but not smooth; this must be resolved by a suitable modification of the Einstein equations in that region of spacetime. Notice that while this was optional in the two-dimensional case, it is compulsory here, since we have argued that the sections *must* be toral; FRW models with toral sections cannot have a section of zero extrinsic curvature if only non-exotic matter is present and if the Einstein equations hold exactly.

A concrete suggestion as to how the smoothing occurs was made in [33], where it was proposed that the structure responsible was the "classical constraint field" proposed by Gabadadze and Shang [38][39]. This leads, in the four-dimensional case, to a Friedmann equation of the form

$$\left(\frac{\dot{\mathrm{a}}}{\mathrm{a}}\right)^2 \;=\; \frac{1}{\mathrm{L}_{\mathrm{inf}}^2} \;-\; \frac{\mathrm{b}\,\varepsilon}{6\,\mathrm{a}^6}\,, \tag{5.2}$$

where we assume that the "constraint field" is significant during a short interval between the creation of the Universe and a subsequent inflationary era characterized by a length scale $\mathrm{L}_{\mathrm{inf}}$, and where $\mathrm{b}\varepsilon$ is a certain constant which may in general be positive or negative[12]. If we wish to create such a universe from "nothing", however, $\mathrm{b}\varepsilon$ is fixed by the requirement that the Euclidean/Lorentzian transition be smooth: this imposes

$$\mathrm{b}\,\varepsilon \;=\; 6/\mathrm{L}_{\mathrm{inf}}^2, \tag{5.3}$$

where we take it that the scale function is equal to unity at the transition. Substituting this into equation (5.2), we obtain an equation which can be solved exactly; the resulting

[11] Of course, a three-dimensional torus with signature $(-\;-\;-)$ has essentially the same geometry as a torus with signature $(+\;+\;+)$.

[12] In the model of Gabadadze and Shang, the spatial sections are flat manifolds-with-boundary, but the idea of the constraint field also works for flat compact sections, assumed here.

JHEP10(2006)029

metric is, in the notation of [33],

$$g_c(6,\,\mathrm{K},\,\mathrm{L_{inf}})_{+---} \;=\; \mathrm{dt}^2 \;-\; \mathrm{K}^2 \cosh^{(2/3)}\!\left(\frac{3\,\mathrm{t}}{\mathrm{L_{inf}}}\right)[\mathrm{d}\theta_1^2 \;+\; \mathrm{d}\theta_2^2 \;+\; \mathrm{d}\theta_3^2]; \tag{5.4}$$

here K is the parameter which fixes the size of the initial torus. Notice that, as t tends to positive infinity, this metric quickly becomes indistinguishable from that of Spatially Toral de Sitter spacetime, $g(\mathrm{STdS_4})_{+---}$. Thus, the spacetime has a spacelike future conformal infinity.

This metric can be expressed in terms of a dimensionless Lorentzian conformal time η_+, defined by

$$\eta_+ \;=\; \frac{\beta}{3}\int_{3\mathrm{t}/\mathrm{L_{inf}}}^{\infty} \mathrm{sech}^{1/3}(\mathrm{x})\,\mathrm{dx}, \tag{5.5}$$

where β is the constant defined by

$$\beta \;=\; \frac{3\,\pi}{\int_0^\infty \mathrm{sech}^{1/3}(\mathrm{x})\,\mathrm{dx}} \;\approx\; 2.5871. \tag{5.6}$$

Notice that the integral in equation (5.5) will converge even if proper time is integrated to infinity. This means that the smoothing of the geometry at the transition point automatically truncates the conformal time to a finite range — we do not have to do this by hand, as we did in the case of pure Spatially Toral de Sitter spacetime. In view of this, we have defined η_+ so that, as in the case of $\mathrm{STdS_4}$ discussed earlier, it is equal to zero at t $= \infty$, and $\eta_+ = \pi$ at t $= 0$; thus η_+ ranges from 0 to π [though this way of putting things reverses the usual direction of time].

If one were able to do the integration[13] in equation (5.5), one would be able to express $\mathrm{t}/\mathrm{L_{inf}}$ as a function of η_+ and so the scale function can likewise be regarded as a function of η_+. Let us define a function $\mathrm{G}(\eta_+)$ by

$$\beta\,\mathrm{G}(\eta_+) \;=\; \cosh^{1/3}(3\mathrm{t}/\mathrm{L_{inf}}). \tag{5.7}$$

Notice that this implies that $\mathrm{G}(\pi) = 1/\beta$. Since the right side of this equation is approximated by $(\mathrm{e}^{\mathrm{t}/\mathrm{L_{inf}}})/2^{(1/3)}$ when t is large, it follows from equation (3.21) that for η_+ close to zero, we have

$$\beta\,\mathrm{G}(\eta_+) \;\approx\; \frac{\pi}{2^{1/3}\,\eta_+}. \tag{5.8}$$

This relation implies that complexifying η_+ necessarily entails complexifying $\mathrm{G}(\eta_+)$; that is, $\eta_+ \to \pm \mathrm{i}\eta_-$ implies $\mathrm{G}(\eta_+) \to \mp\mathrm{i}\mathrm{G}(\eta_-)$, where η_- is Euclidean conformal time, which takes its values in $[-\pi\,,\,0)$, with the transition at $-\pi$ and Euclidean infinity at $\eta_- = 0$.

It follows that if we write $g_c(6,\,\mathrm{K},\,\mathrm{L_{inf}})_{+---}$ in the form

$$g_c(6,\,\mathrm{K},\,\mathrm{L_{inf}})_{+---} \;=\; \mathrm{G}(\eta_+)^2\,[\mathrm{L_{inf}^2}\mathrm{d}\eta_+^2 \;-\; \beta^2\,\mathrm{K}^2\,\{\mathrm{d}\theta_1^2 \;+\; \mathrm{d}\theta_2^2 \;+\; \mathrm{d}\theta_3^2\}], \tag{5.9}$$

then its Euclidean version is just

$$g_c(6,\,\mathrm{K},\,\mathrm{L_{inf}})_{++++} \;=\; \mathrm{G}(\eta_-)^2\,[\mathrm{L_{inf}^2}\mathrm{d}\eta_-^2 \;+\; \beta^2\,\mathrm{K}^2\,\{\mathrm{d}\theta_1^2 \;+\; \mathrm{d}\theta_2^2 \;+\; \mathrm{d}\theta_3^2\}]. \tag{5.10}$$

[13] It can of course be done, in terms of hypergeometric functions; the result is not useful, however.

JHEP10(2006)029

This can of course be written as

$$g_c(6,\,K,\,L_{\rm inf})_{++++} \;=\; d\Phi^2 \;+\; K^2 \cosh^{(2/3)}\!\left(\frac{3\,\Phi}{L_{\rm inf}}\right)[d\theta_1^2 \;+\; d\theta_2^2 \;+\; d\theta_3^2], \qquad (5.11)$$

where Φ runs from $-\infty$ to zero. This is indistinguishable from $g(H^4/\mathbb{Z}^3)_{++++}$ for sufficiently large negative Φ. Thus $g_c(6,\,K,\,L_{\rm inf})_{++++}$ and $g_c(6,\,K,\,L_{\rm inf})_{+---}$ interpolate between the two metrics we wish to join, *but this is done smoothly.*

The full metric, combining the Euclidean and Lorentzian versions of the metric on a manifold of topology $\mathbb{R} \times T^3$, is now

$$g_c(6,\,K,\,L_{\rm inf};-\pi \,\leq\, \eta_- \,<\, 0)_{++++} \;\longrightarrow\; g_c(6,\,K,\,L_{\rm inf};0 \,<\, \eta_+ \,\leq\, \pi)_{+---}. \qquad (5.12)$$

The Euclidean-to-Lorentzian transition is at $\eta_- = -\pi$ and $\eta_+ = \pi$, that is, at $t = \Phi = 0$. The two spaces are joined along a torus of radius K in a way such that the scale factor of the full metric is infinitely differentiable.

Note that the angular interpretation of conformal time seems particularly natural in this case, since no truncations have to be performed by hand. If one identifies $\eta_+ = 0$ with $\eta_- = 0$, then the conformal compactification is fully toral: it is a four-torus. The inflationary length scale $L_{\rm inf}$ determines the radius of one circle, while K determines the radius of the other three. [Strictly speaking, in conformal geometry only the ratio $K/L_{\rm inf}$ is a well-defined parameter here, so one should say that the initial size of the Universe measured in inflationary units is what fixes the shape of the conformal torus.]

Of course, the suggestion that the Gabadadze-Shang constraint field is responsible for the smoothing is just one possibility; there are others; the main point is that the smoothing can be done in a physical way. It seems reasonable, however, to assert that the metric given in (5.12) is the simplest possible smooth model of a four-dimensional Euclidean space of the OVV type giving rise to an accelerating Lorentzian cosmological model.

6. Conclusion

We can summarize as follows. The OVV wave function is formulated on [a partially compactified version of] hyperbolic space, $H^2/\mathbb{Z}$. By means of a simple extension of the concept of complexification, we have been able to explain how to realize the idea of Ooguri et al that this space has two Lorentzian interpretations, one [equation (3.17)] like anti-de Sitter, the other [equation (3.20)] like de Sitter spacetime. This idea works in all dimensions, but, in dimensions above two, it *only* works in the case where the transverse sections are flat tori. The global structure of the three-dimensional sections of our Universe is thus *emergent*, in Hartle's [12] sense, from the OVV formalism.

The flatness of the spatial sections requires that Einstein's equations be corrected near the creation event, so that the Euclidean/Lorentzian transition can be smooth; we have suggested a concrete way, based on the ideas of Gabadadze and Shang [38][39], whereby the smoothing can be achieved in a physical manner.

Obviously a great deal remains to be done. First, one must indeed extend the OVV theory to four dimensions. The first step would be to replace $H^2/\mathbb{Z}$ with its natural higher-dimensional version, the partial compactification $H^4/\mathbb{Z}^3$. Ooguri et al embed $H^2/\mathbb{Z}$ in

JHEP10(2006)029

IIB string theory by considering a background of the form $(\mathrm{H}^2/\mathbb{Z}) \times \mathrm{S}^2 \times \mathrm{CY}$, where CY denotes some Calabi-Yau manifold. One way to embed $\mathrm{H}^4/\mathbb{Z}^3$ in string theory — or, rather, M-theory — might be through a compactification of the form $(\mathrm{H}^4/\mathbb{Z}^3) \times \mathrm{FR}$, where FR denotes a [singular] Freund-Rubin space of the kind studied in [40]. Extending the OVV ideas to spaces of this kind is a challenging problem. If it can be done, the next step would be to try to understand the consequences of smoothing the Euclidean/Lorentzian transition in this context. One would then be able, by means of a complexification of the kind suggested here, to see what the OVV theory predicts regarding the nature of four-dimensional accelerating cosmologies.

We argued above that it is natural to think of η_+ and η_- as angular variables: the identification of the two spaces at the Euclidean-to-Lorentzian point is then expressed by the familiar fact that $-\pi$ and $+\pi$ refer to the same point in plane polar coordinates. If we take this to its logical conclusion, then we should *also* identify $\eta_- = 0$ with $\eta_+ = 0$. In other words, Euclidean conformal infinity is just Lorentzian future spacelike infinity, approached from the "other side". In this case the full conformal compactification of the combined space with metric (5.1) or (5.12) has the topology of a four-dimensional torus. This way of thinking could possibly be of interest in connection with ideas about holography at future spacelike infinity in accelerating cosmologies[14]. In particular, it might help us to understand how a necessarily Euclidean conformal field theory at future spacelike infinity can be dual to Lorentzian physics. The idea would be that the duality is effective through the Euclidean-to-Lorentzian transition, by going the "long way" around the circle, clockwise from $\eta_- = 0$ to $\eta_- = -\pi$, through the Euclidean-to-Lorentzian transition there, from which the Universe evolves in Lorentzian conformal time from $\eta_+ = \pi$ back to future spacelike infinity at $\eta_+ = 0$.

The minisuperspace construction considered by Ooguri et al leads to a prediction that the most probable geometry is flat space; this is reminiscent of the conclusion, derived from the Hartle-Hawking wave function, that the most probable value of the cosmological constant is zero [42]. In the latter case it has been argued convincingly in [6][7][8] that the wave function needs to be modified in some way that will involve going beyond the most basic minisuperspace constructions. We have seen here that, in the OVV case, one has to go beyond the most basic minisuperspace models merely to obtain a smooth Euclidean/Lorentzian transition, leading to a metric which could resemble the one given in (5.12) above. Perhaps this geometry will be useful in an attempt to extend the ideas of [6][7][8] to the OVV wave function, so that more reasonable predictions can be made. The obvious first step would be to try to predict the value of the parameter K in (5.12), to see whether the modified wave function predicts a physically acceptable value for the initial size of the Universe. A value close to the string length scale would be particularly interesting: for that would implement T-duality for the circles constituting the initial three-torus, in the sense that no circles of radius smaller than the string scale would ever exist.

[14]See in particular the discussion of this kind of holography in [41].

JHEP10(2006)029

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